

# A distribution on the simplex of the Generalized Beta type

Monique Graf

Institut de Statistique, Université de Neuchâtel

Elpacos Statistics, la Neuveville, Switzerland; *monique.p.n.graf@bluewin.ch*

## Abstract

The multivariate Generalized Beta distribution of the second kind (MGB2) has been recently derived by Yang et al. (2011). It is a mixture of  $D$  independent generalized gamma variables with an inverse gamma mixing distribution. Starting from a random vector  $\mathbf{Y}$  following the MGB2 distribution, we derive the distribution of the closed random vector  $\mathbf{U} = C(\mathbf{Y})$  and call it the simplicial Generalized Beta (SGB). The density of the distribution is given. Some of its properties are derived and exemplified. It is shown that the SGB distribution is not of the Liouville type (Rayens and Srinivasan, 1994) and is not neutral either (James, 1975). Nevertheless it contains the scaled (or shifted) Dirichlet distribution of Monti et al. (2015, 2016) as a special case. The moments of ratios of parts are derived. Moments of parts can be obtained indirectly by Taylor linearization. Conditional moments given a sub-composition are computed. They give a way to impute missing parts when knowing a sub-composition only. Maximum likelihood estimators of the parameters are obtained and the technique is applied to the Arctic lake example also considered by Monti et al. (2015), but contrary to these authors, who modeled the shape parameters, here the scales are made dependent of the auxiliary variable.

**Key words:** Dirichlet distribution; Generalized Beta distribution of the second kind; simplicial Generalized Beta; maximum likelihood estimation; imputation.

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